In this project, undergraduates would study the question of existence of a smooth closed geodesics on compact Riemannian 2-orbifolds using theoretical and computational techniques along with computer visualization. Roughly speaking a Riemannian orbifold is a metric space locally modeled on quotients of Riemannian manifolds by finite groups of isometries. The 2-orbifolds we consider are orbifolds whose underlying space is a compact 2-manifold without boundary. One can think of such Riemannian orbifolds as surfaces with some distinguished singular cone points whose neighborhoods are isometric to a quotient of the unit disc with some metric by a finite cyclic group of isometries fixing the center of the disc. The 2-orbifolds we consider fall into two categories. The first case is when the underlying space of the orbifold is simply connected (in the usual topological sense), that is, the underlying space of the orbifold is the 2-sphere. These 2-orbifolds are examples of what are commonly referred to as teardrops and footballs. The second class of 2-orbifolds are those whose underlying space is not simply connected in the usual sense.

To better understand the context of the proposed project, one should recall the classical theorem of Fet and Lyusternik: On any compact Riemannian manifold there exists at least one closed geodesic. The essential tool in proving this classical result is to develop a process of curve shortening. Theoretical existence of such closed geodesics has been investigated using techniques from topology, geometry and dynamical systems, but a complete analysis on many types of specific Riemannian 2-orbifolds is still lacking. It is known now, however, as a result of previous NSF REU work done at Cal Poly (http://www.calpoly.edu/~jborzell/Publications/Publication%20PDFs/oninfinitely.pdf), that on spherical 2-orbifolds of revolution there exist infinitely many geometrically distinct closed geodesics.

Using a symbolic computation package such as Maple, undergraduates would formulate and numerically solve the equations governing the geodesic flow on specific Riemannian 2-orbifolds, such as those that arise as surfaces of elliptical revolution or those that admit flat structures. Additional use of the visualization capabilities of Maple would lead them to conjectures on qualitative aspects of geodesics on Riemannian 2-orbifolds and corresponding proofs.

Student background should include multivariable calculus, linear algebra and differential equations.