Let R be the polynomial ring in n variables over a field k. We can measure the complexity of (nice) ideals in R by considering their Hilbert functions (which gives the dimensions of each degree piece of the ideal as a vector space over k), or more carefully, by considering their graded Betti numbers. Roughly speaking, an ideal's graded Betti numbers count the number of generators of the ideal, the number of generators of the module obtained by allowing the ideals generators to interact, and so on iteratively. It's really a matter of counting dimensions of certain vector spaces arising from the ideal in question.

It is known how to find ideals with big graded Betti numbers. It is not, on the other hand, well understood how to find ideals with small graded Betti numbers (or how to tell when a potential set of small graded Betti numbers actually occurs "in nature"). Tony Geramita has suggested a possible combinatorial (and hence, explorable with a computer) restriction which might categorize small graded Betti numbers, and recent work of Eisenbud (et. al.) makes it more likely that small graded Betti numbers might be identifiable in particular instances. Thus the time is ripe to explore Geramita's idea, and either find a counter-example, or amass enough evidence to direct working on a proof (the ideals in question have so much inherent structure, that there is hope that such a proof is within reach).

The work would be early in the summer (starting right after finals).

Applicants should have had 306, and 481/482 (this not absolutely necessary, but would complicate things a bit without). Some programming skills are necessary.