

Mathematics Colloquium

Paradox of Individuals and Formal Language of Analysis

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Friday, October 28, 2005
4:10 p.m. - 5:00 p.m.
Science North Building 53, Room 202

Abstract

According to the Axiom of Extensionality the only set without members is the empty set. Yet, unless we are proving the consistency of the axioms, we prefer to think of the reals as **individuals or abstract points** rather than sets. (After all who wants to deal with propositions such as $e \cap \pi = \emptyset$?). In linear algebra the set of individuals is $\mathbb{R} \cup V$, where \mathbb{R} is the set of the real numbers and V is the vector space under consideration. In point set topology the set of individuals is $\mathbb{R} \cup X$, where (X, T) is a topological space. In functional analysis we often treat $\mathbb{R} \cup \mathcal{H}$ as the set of individuals, where \mathcal{H} is an “abstract Hilbert space”, etc. The elegance of modern mathematics is due partly to the involvement of the individuals other than the empty set. We call this the **paradox of individuals** because individuals do not have obvious legitimacy in set theory. In this talk we shall explain how this paradox can be resolved and the individuals can be rigorously legitimized. This work was started during this summer’s REU program jointly with Cal Poly **students Diana Mircheva and Scott Maccarone**. The speaker would like to thank both of them for the very stimulating discussions.